Newton’s Mathematical Proof of Elliptical Orbits

“If I have seen farther, it is by standing on the shoulders of giants”

-Isaac Newton (in a Letter to Robert Hooke, 1676)

Introduction

The historical roots of modern astronomy can be traced back to the first, ancient observers of the sky – those who relentlessly pondered about the fundamental laws and mechanisms of the universe. In documented histories of science, there was much speculation about the fundamental laws of nature, but only few great minds have come to realize them. The roots of our modern astronomy and astrophysics, as we know it today, evolved after many centuries of scientific speculation, both imagined and reasoned, about the movements and nature of both the terrestrial and celestial realms – as the two were found to be connected under a unified physical theory. Major refinements to our modes of reasoning, methodologies of inquiry and technologies, played a key role in our ability to understand, describe and predict the observable patterns found in the universe. It was not until the birth of modern science – a body of techniques for investigating phenomena, consisting in systematic observation, experimentation, and calculation – that major advancements could be made in our understanding of the cosmos. And one cannot give an accurate telling of this story without mentioning the role of mathematics – after all, it was the mathematization of Nature, beginning with the early Pythagorean school of thought, that we could write down the fundamental laws of Nature in the first place.

The old conviction that Nature is, at its core, an expression of simple mathematical principles continued well into the modern era, an idea that is still prevalent in modern scientific research. This paper will select episodes in this history by focusing primarily on the first mathematical proof for elliptical planetary motion, beginning with Galileo and culminating with Sir Isaac Newton. More specifically, this paper will discuss the mathematical details found in Newton’s monumental work, *Philosophiæ Naturalis Principia Mathematica* (1687) – a publication that is widely considered one of the most important contributions to modern science. Here, I will outline some features of Newton’s mathematical proof of Kepler’s three laws of planetary motion. The objective of this paper is to instill within the reader a deeper appreciation for the scientific process and the mathematical ingenuity that is so essential in understanding a mathematical universe.
The Early Pythagoreans and the Mathematization of Nature

The idea that Nature could be understood using mathematics echoes the views of the ancient Pythagoreans. The early Pythagorean school of thought was an ancient Greek philosophical tradition that was heavily influenced by mathematics and mysticism. The Pythagoreans of the inner circle believed that all of Nature was number – that is, all seemingly chaotic phenomena in the natural world was a manifestation of number relationships, more specifically, as ratios of integers (with the exception of $\sqrt{2}$, which could not be expressed as a ratio of whole numbers) (Kline, 189). Two major ideas/attitudes emerged from the Pythagorean school of thought, namely that: (1) Nature could be understood through careful mathematical investigation, as Nature itself was a manifestation of number; and (2) hidden from plain view are fundamental, mathematical laws by which Nature operates. In the context of astronomy, the heavenly spheres were thought to be related by whole number ratios of pure musical intervals. According to the Pythagoreans, the motion of the planets created a musical harmony that can only be imagined with a mathematically trained mind.

The Pythagorean idea, that Nature was fundamentally mathematical, carried on through the ages and became a propelling factor in the advancement of astronomy and physics. This, in fact, was the philosophical outlook that prevailed through many centuries, as evidenced in Johannes Kepler’s publication *Harmonices Mundi* (1619), until Galileo had formed both the methodological and mathematical foundations of modern science – a new lens through which to conduct experiments about Nature. Before this could happen, however, many centuries of speculation, which resulted in a vast array of competing theories and models about the structure of the observable universe – competing theories that had little to no factual basis in Nature – had to be discarded before a new model, one that had accurately represented Nature, could be developed. To fully appreciate the fruits of the modern science (in the context of astronomical physics), first set out by Galileo, it is important to first appreciate our early efforts to comprehend the cosmos. This paper will begin with a brief documentation of this history, beginning with the ancient Greek philosophers, and later culminating with the scientific advancements of Galileo, Kepler and Newton.
Early Pictures of the Universe

Based on archaeological evidence dating back to 1200 BC, the earliest peoples have contemplated the vastness of the universe and have recorded, in illustration, their understanding of the cosmos (refer to Fig.1: The Nebra Sky Disk). They had some notion about the dynamical changes in universe. They catalogued the stars and could make the distinction between “wandering” bodies (i.e. planets) and the constellation of stars – background bodies that stayed fixed in their relative positions across the sky.

Fig. 1: The Nebra Sky Disk (1,200 BC)

Of course, the early peoples had no way of making any further distinctions among the stars in the night sky besides that fact that some stars were more luminous and/or travelled faster across the night sky than others. But one thing we do know is that they had some notion about cyclical time (repeating pattern of day and night cycles and the link between seasonal changes and orientation of the sun and moon), order (regularity in cycles was somewhat orderly and predictable), and structure (the spatial arrangement of the celestial bodies and their relative movement across the sky) in the universe. It took more than a thousand years before people would employ more sophisticated technology for making sense of the universe. The next chapter in this story leads us to the ancient Greeks, the first people who proposed a mathematical structure to the universe.

Ancient Greek Cosmology

The ancient Greeks had excelled in philosophy, astronomy and science. One major area of speculation was in cosmology. Ideas about the creation of the world and the order and
structure of the observed universe were important for their understanding of both the fundamental nature of the universe and man’s place in the world. Similarly, the pre-Socratic philosophers had imagined Nature as a manifestation of a more fundamental reality. Democritus (c. 535 - 475 BC) was known for his atomism – a philosophy that hypothesized the existence of atoms, the basic building blocks of Nature. How these atoms were arranged and made to bind together to form structure, and under what forces, was unknown at the time. But what is significant to note here is that the ancient Greeks began thinking about the underlying structures of Nature and that of the greater cosmos. The fact that Nature is designed, operating under basic physical principles (i.e. sometimes ones that cannot be seen with the naked eye), hence the fundamental laws of nature, tells us that the early Greeks had some primitive ideas about the mechanisms underlying natural phenomena (like the movement of the stars and planets, for example). The Pythagoreans, for example, had postulated a fundamentally mathematical substratum in Nature. One theory that survived from the Pythagorean tradition was the idea that the heavenly spheres (i.e. stars and planets) moved according to specific mathematical proportions, or musical harmonies (see Fig. 2 below).

Fig. 2: The Pythagorean Music of the Spheres

We see this same philosophy embedded in the work of German astronomer Johannes Kepler (1571-1630), when he first proposed the three laws of planetary motion. With respect to astronomy, a wide array of ancient Greek planetary models existed. Showcased here are the ancient Greek cosmological models of Anaximander (c. 610 - c. 546 BC), Aristarchus (c. 310 – c. 230 BC), Aristotle (384–322 BC), and Ptolemy (c. AD 100 – c. 170) respectively:
Fig. 3: Anaximander’s Cosmology (Geocentric Model, aerial and side views)

![Diagram of Anaximander's Cosmology]

Figure 1
A map of Anaximander's universe

Figure 2
Anaximander's universe in summer, by day

Figure 3
Anaximander's universe in winter, by night
Anaximander’s cosmological model (see fig. 3) postulated an Earth-Stars-Moon-Sun system, where the spacing of each ring, each having a spacing of one Earth diameter, followed a specific numerical (i.e. Earth diameter) formula. Anaximander goes as far as to account for the change in seasons by simply adjusting the sun ring for the summer (sun being at its highest point in the sky) and winter months (sun being at its lowest point in the sky). Note that the background constellation of fixed stars circle around the Earth within a spherical shell. The sun, being at the furthest point away from the Earth, emits light out of an aperture that rotates around the stationary Earth.

Fig. 4: Aristarchus’ Cosmology (First Heliocentric Model)

Aristarchus’ cosmological model (see fig. 4) is quite different from Anaximander’s in that he places the sun at the center of the universe and has the Earth orbit around the sun along with the moon, which is made to rotate in a two-body dynamical system with the Earth.
Aristotle’s cosmological model (see fig. 5) places a stationary Earth at the center of the cosmos while incorporating a sphere at the outer edge for God, the supreme mover. According to Aristotle’s’ philosophy, all heavy objects fell towards a heavy Earth. Both the planets and background stars were of a lighter substance as they revolved in circular orbits far away from the Earth-moon system. Aristotle incorporates a supreme mover in his model to account for the first cause of all motion in the universe.
**Fig. 6: Ptolemaic Cosmology**

Ptolemy’s cosmological model (see fig. 6) places the Earth on a deferent some distance away from the center of the universe and has the planets move around the Earth. The retrograde motion of Mars motivated Ptolemy to create the epicycles.

In retrospect, these models lead us to conclude that mathematics played a central role in developing pictures of the underlying mechanisms of the cosmos. Circle geometry is obviously the common denominator in these models, but with closer inspection we can see more elaborate details in their reasoning. In Aristarchus’ model, for instance, we see, for the first time, differentiation between planets and the background stars – the five identifiable planets being: Mars, Venus, Mercury, Jupiter, and Saturn. Does this suggest that the ancient Greek astronomers thought of the universe as a grand, mathematically organized structure? There is evidence to suggest this, which reinforces the Pythagorean idea that Nature is fundamentally mathematical. However, as convincing this conjecture might sound, one must not over generalize the facts. We can only suggest that mathematical entities such as number...
and shape played a major role in formulating their theories about celestial dynamics. Below (see fig. 7) we see the track of Mars across the night sky. Careful observers of the night sky were well aware of Mars’ challenging orbit. As Ptolemy knew that nothing in Nature can move forward and backward, he created circular epicycles to account for this motion. It was a novel idea at the time, and it did de-mystify the orbit of Mars. And although Ptolemy had provided a comprehensible model of the cosmos, his epicycles faded into history as, later, the famous Polish astronomer and mathematician, Nicolaus Copernicus (1473-1543), demonstrated that an even simpler, heliocentric model can account for the retrograde in the orbit of Mars (see fig. 8, simplified Sun-Earth-Mars system).

**Fig. 7: A Time Lapse Retrograde of Mars**

![A Time Lapse Retrograde of Mars](image)

**Fig. 8: Retrograde of Mars in Copernicus’ Heliocentric Cosmological Model**

![Retrograde of Mars in Copernicus’ Heliocentric Cosmological Model](image)
Copernicus’ cosmological model suggests two things: (1) If Earth were to travel faster than Mars in its orbit around the Sun, then retrograde motion would only seem as an illusion to an observer on the Earth; and (2) epicycles are not necessary if a simpler explanation can be provided.

According to the early Greek philosophers, the fundamental laws of nature were the core building blocks of reality. What the ancient Greeks taught us is that truth is not always obvious and that trained human reason can lead us to understand the underlying mechanisms of Nature. The early Greeks were the first people who applied mathematics and logical reasoning to understand how the universe was designed. Modern science would follow in their footsteps, “…the Greeks fashioned a conception of the universe which has dominated all subsequent Western thought. They affirmed that Nature is rationally and indeed mathematically designed” (Kline, 187). For more than one thousand years, Scientists, philosophers and astronomers would continue to develop their understanding of the cosmos. With the advent of modern science, the dynamical principles of the universe would become less mysterious to the inquiring mind.

The Beginning of a New Science

Many scholars attribute the beginning of modern science to the famous Italian scientist/astronomer/mathematician, Galileo Galilei (1564-1642). He was the first to challenge, and later replace, Aristotelian intuition about dynamical motion. Galileo offered a new way of investigating Nature. He was known for his ingenious experimental methods and attention to detail. He did not readily accept popular theories about the physics of motion. On the contrary, he consistently pushed the limits of his own understanding about physics. As Galileo was after the underlying mechanism of motion itself, he was not satisfied with simple, common-sense fabrications about the physics of motion – a body of knowledge about the world that was passed down since the time of Aristotle. Nor was he satisfied with the Scholastic tradition of formulating theories without reference to experimental data. It was this new way of studying Nature – by methodically reconsidering our common-sense notions about the physics of motion, and holding them up to the scrutiny of experimentation – that led to a new physics of Nature (i.e. the foundations of classical mechanics). More significant was his search for the mathematical principles of motion.

In 1623, Galileo had published a book entitled Il Saggiatore (The Assayer), considered a milestone in the history of science, which described his scientific methods and highlighted his conviction that Nature was written in the language of mathematics (see fig. 9 below).
And not just Nature itself, but science too was written in the language of mathematics, since it involves the analysis of empirical and measurable evidence.

“Nature is written in that great book which ever is before our eyes -- I mean the universe -- but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it; without which one wanders in vain through a dark labyrinth.”

Galileo was convinced that natural phenomena, in the terrestrial realm, could be properly understood using two primary ingredients, namely; applied experimentation and mathematical analysis. Galileo believed that once natural phenomena is translated into mathematical language, deductive reasoning can be applied to draw out indubitable conclusions about their properties (and later re-checked against repeated experimentation). If results (i.e. conclusions) from repeated experimentation contradicted the original hypothesis, then the hypothesis would be discarded and a new one formulated. Theories about natural phenomena, according to Galileo, are considered scientifically valid if and only if they fit the observed data. Theories that do not fit empirically consistent data must be discarded to ensure scientific integrity and to prevent discrepancies among the wider scientific community. When Galileo pondered the
fundamental laws of Nature, and the means to discover them, we see in his professional life that there was no room for dogma and unreasoned speculation in the pursuit of exact knowledge. Galileo knew very well that only a trained mind, following diligently a method for studying Nature, could uncover the secrets of Nature. Galileo set the standard for scientific enquiry, and encouraged other scientists to invest the same attention and care with their pursuit. A new scientific ethos was modelled, and this became the staple of modern science – it was the sure way to achieve lasting truths in the new physics of the world.

**Galileo’s Scientific Method**

What set Galileo apart from his contemporaries, and more significantly from his ancient forebears, was his employment of a systematic method of experimentation. He postulated physical theories only when his hypotheses were confirmed through observed, experimental data. This method was useful for Galileo as it guaranteed precise knowledge about the laws of Nature. Galileo’s contributions had dramatically accelerated our understanding about the universe, and his investigative methods served as a template for doing proper science. Galileo’s publication *De Motu* (On Motion) – a dynamical study on falling bodies that contradicted Aristotelian theories of motion – is testament to his intellectual superiority. The cornerstone of modern physics, however, was Galileo’s publication *Dialogues Concerning Two New Sciences* (1638), which described the mathematical laws of accelerated motion governing falling bodies. Soo revolutionary was Galileo’s scientific work that Albert Einstein once wrote: “Propositions arrived at purely logical means are completely empty as regards reality. Because Galileo saw this, and particularly because he drummed it into the scientific world, he is the father of modern physics – indeed of modern science” (Einsten in Hawking, 397).

**Galileo’s Incline Experiment**

Galileo is known for his masterful execution of experiments and his precision in analyzing experimental data. He had conducted detailed experiments on the physics of: falling bodies, pendulums, and inclined planes. He concluded, from his study on falling bodies, that bodies fall according to a precise mathematical formula. Galileo had also discovered that bodies, close to the surface of the Earth, fall with a constant acceleration. But how exactly did Galileo reach this conclusion? He knew that if he wanted to accurately study the physics of falling bodies, he had to “slow down” the experiment to ensure accuracy in his readings (this proved more difficult when dropping objects straight down. Galileo solved this problem by dropping ball bearings down inclined planes at varying degrees of inclinations (see Fig. 10).
As Galileo investigated the data, he noticed that the ball bearing had covered distances that were precisely at squares to each time interval. After $t = 1 \text{ sec}$, the ball had fallen one unit down. After the $t = 2 \text{ sec}$ the ball had fallen a total distance of four units. After the $t = 3 \text{ sec}$ the ball had fallen a total distance of nine units.

His research implied that bodies fall in a predictable way, expressed as a functional relationship between time and distance, $d = kt^2$, where $k = \frac{-1}{2}a$. Galileo found that the acceleration of the falling ball bearing was constant. Hence, Galileo concluded that bodies fall according to the proportion $d \propto t^2$. Galileo states, in his Dialogues Concerning Two New Sciences, that: The space described by a body falling from rest with a uniformly accelerated motion are to each other as the squares of the time-intervals employed in traversing these distances (Galileo, Theorem II Proposition II, in Hawking 531). One important question remained: What is the
nature of the acceleration that acts on all falling bodies? In attempting to answer this question, Galileo reasoned that motion is caused by impressed forces. He then asked: Is it possible that the phenomenon of acceleration could be linked to an inherent force found in Nature (i.e. a force that is causing the acceleration of bodies towards the center of the Earth)? Two basic facts about dynamics were believed to be true since the time of Aristotle: (1) The greater the impressed force is the greater the speed of the body (i.e. $F \propto V$); and (2) All bodies that are set in motion eventually slow down and seek the center of the earth because their impressed forces dissipate from the body with time. Galileo was not satisfied with these descriptions about the nature of motion. He knew that there was an underlying mechanism in Nature that caused bodies to fall with uniform acceleration, just as his experimental evidence had suggested. As Galileo reasoned about the link between force and velocity, he performed a thought experiment (see Fig. 12): If a body travels with uniform motion, in the absence of external forces (like friction), and is allowed to travel to infinity, how could one tell if a force is acting on the body?

**Fig. 12: Galileo’s Thought Experiment: Uniform Motion**

His immediate answer was that the force would remain undetectable. According to Galileo’s **Principal of Inertia**, a body in motion will continue its motion so long as no factor disturbs that motion, a body will continue moving at the same velocity forever. Galileo also adds another detail: “...the distances traversed by the [uniformly] moving particle during any equal intervals of time, are themselves equal” (Galileo in Hawking 515) (see fig. 12). And so, he extended this idea and asked himself: What would serve as evidence for an external force acting on a body moving with uniform motion? He then realized that the only way to detect if a force is acting on a body is if three things happen: (1) the body suddenly slows down (suggesting that a force was acting in the opposite direction of its original path); or (2) the body speeds up in the same direction of its original path (suggesting that a force had pushed the body in the same direction of its original path); or (3) the body suddenly changes direction (suggesting that a force had
acted at some angle to the body). Galileo immediately found the missing clue. Force, for Galileo, causes a change in velocity, either to speed it up or to slow it down! Therefore, unlike Aristotle’s common-sense reasoning, force is not linked to velocity itself but rather with the change of velocity (i.e. $F \propto \Delta V$). This was a revolutionary idea because it implied that: (1) force is mathematically related to acceleration; and (2) the presence of a force can only be detected if and only if there is either a change in velocity (acceleration or deceleration) or change in direction of an object that is either at rest or in uniform motion. He wrote in his Two New Sciences:

“... any velocity once imparted to a moving body will be rigidly maintained as long as the external cases of acceleration or retardation are removed, a condition which is found only in inclined planes; for in the case of planes which slope downwards there is already a present a cause of acceleration, while on planes sloping upward there is retardation; from this it follows that motion along a horizontal plane is perpetual; for, if the velocity be uniform, it cannot be diminished or slackened, much less destroyed.”

(Galileo in Hawking, 564)

Galileo had concluded that the phenomenon of natural acceleration was inherent (i.e. built in) in space and that it acted on all bodies. Space was conceptualized as exerting a downward “pulling” force towards the center of the Earth – a force that explained and accounted for both accelerated motion, so-called violent motions, and with projectiles.

The fact that falling bodies abide by a simple mathematical relationship is truly fascinating! The fact that Galileo had conceptualized this phenomena as a simple mathematical relationship speaks to his ingenuity and talent as scientist. Galileo discovered the underlying mechanism of acceleration by making explicit the intimate relationship between time and motion. The practice of science had dramatically changed in the sense that the fundamental laws of nature could be discovered through reason and careful experimentation. More important is the idea that the fundamental laws of nature exhibited simplicity and that it could be expressed mathematically.

Galileo’s contributions led to much progress in the astronomical sciences, especially in the study of kinematics (with Kepler) and dynamics (with Newton). The next chapter of scientific discovery draws heavily on Galileo’s understanding of the relationship between force, space, and time.
Kepler Proposes Three Laws of Planetary Motion

Continuing in this tradition of combining experimentation with mathematical analysis (i.e. Data Analysis) came the major scientific contributions of German Astronomer Johannes Kepler (1571-1630) and English scientist Isaac Newton (1643-1727). I wish to expand first on Kepler’s work. Kepler had conducted an important mathematical study of the astronomical charts, namely, the *Rudolphine Tables*, inherited from his mentor Tycho Brahe (1546-1601), a famous Danish imperial astronomer who was known for his observational accuracy and precision in naked-eye astronomy. Following a careful analysis of the astronomical data – information that spanned 30 years of Brahe’s career – Kepler reached new conclusions about the nature of planetary motion. In Brahe’s eyes, Kepler’s had demonstrated both the analytic and mathematical skills required to understand the astronomical data, essential skills that Brahe lacked in his own study. It was documented that Brahe had used Kepler’s efforts to support his own view about the universe, namely a geocentric model of the observable universe. However, Kepler was in disagreement as his observations had led to a different conclusion altogether – one that showed clear evidence supporting a heliocentric model of the universe. Once Brahe had died, Kepler continued to study his astronomical charts and came up with three major laws of planetary motion.

While Isaac Newton was forming theories about dynamic motion by 1666, the German astronomer and mathematician Johannes Kepler had proposed three laws of planetary motion, which turned out to be an accurate description of how the planets moved in the celestial realm. Kepler’s contributions can be summarized in his three laws of planetary motion. They were the first mathematical attempt at understanding the physical laws that govern planetary dynamics. These laws are namely; (1) *planets move in elliptical orbits with the sun at one focus*; (2) *planets sweep out equal areas in equal intervals of time*; and (3) *the cubes of the mean distance of the planets from the sun are proportional to the squares of their periods of revolution* \((T^2 \propto R^3)\). It is important to note that Kepler’s laws were developed at different times in his career. His first two laws were published in 1609 in his book *Astronomia Nova (New Astronomy)*, and his third law was later published in 1618, approximately ten years after his *New Astronomy*, in his *Harmonices Mundi (Harmonies of the World)*. Kepler’s study of the orbit of Mars had major implications for the astronomical sciences. Kepler eventually discovered how planets orbited in their trajectories around the sun. Kepler has conjectured that there was a force emanating from the sun, one that would keep the planets moving in their respective orders and harmonies. As his ideas were quite lose and unsupported by any mathematical/scientific proof, any conjecture of this magnitude would have to wait for Newton to discover why the planets moved in this fashion. Let us now revisit Kepler’s three laws and look at them in more detail.
1st Law: Planets move in elliptical orbits with the sun at one focus

Breaking away from tradition, of the many centuries of astronomical inquiry, which placed its faith in the circularity of planetary orbits – as perfect circular motion was reserved for celestial phenomena – Kepler rejected circles altogether and adopted a new model of planetary motion. This new model supported the heliocentric planetary model that Nicholas Copernicus, 16th century Polish priest and mathematician, had first proposed in his landmark publication *De Revolutionibus* (1543). In Kepler’s publication *On the Motion of the Planets* (1609), he had both empirical and mathematical evidence to propose a new geometry for planetary motion. Kepler had concluded, following an eight-year study of the orbit of Mars – an orbit that Brahe had considered to be the least circular – that planets exhibited elliptical trajectories around the sun. According to this model, the planet Earth is closest to the sun in January and farthest from it in July. If not for the ancient Greek mathematicians (i.e. Euclid, Apollonius, and Archimedes), who had first introduced and studied the properties of the ellipse (among other conic sections), Kepler would have been faced with a daunting task to abstract the proper path from a wide range of numerical data (Kline, 329).

Fig. 13: Elliptical Planetary Orbits for a 2-Body System
2nd Law: An imaginary line connecting any planet to the sun sweeps out equal areas in equal intervals of time

Kepler’s second law, or law of equal areas, states that the line drawn from the sun to a planet sweeps out equal areas in equal times. Kepler’s second law, together with his first law, were published 1609 in his publication *Astronomia Nova* (new astronomy). The implication of the second law of planetary motion is that planets move at different speeds along their respective orbits (Kepler in Hawking, 646). According to the second law, a planet must, by necessity, travel faster the closer it comes to the sun than when it is at its farthest point away from the sun (as the planet moves further away from the sun, the planet begins to slow down). This ensures that the areas of the arcs swept out by the planet during the same increment of time remains equal. The idea that planets orbit around the sun at different speeds challenged uniform circular motion in Aristotelian dynamics.

**Fig. 14: Planets sweep out equal areas in equal times**
3rd Law: The cubes of the mean distance of the planets from the sun are proportional to the squares of their periods of revolution)

In addition, Kepler had worked out a new mathematical relationship, in his publication *The Harmonies of the World* (1619), that states a precise mathematical relationship between the periodic times of planets to complete their orbits around the sun and their average distance from the sun. Kepler called this the Harmonic Law as it supported his idea that the movement of the planets created musical harmonies according to specific proportions – these proportions had a special place in Kepler’s cosmological model. After long analysis of Brahe’s observations – one that took him seventeen years to complete – Kepler had reached the grand conclusion that attracted the most notable minds in science: “...I first believed I was dreaming and was presupposing the object of my research among the principles. But it is absolutely certain and exact that the ratio which exists between the periodic times of any two planets is precisely the ratio of the $\frac{3}{2}$th power of the mean distance” (Kepler in Hawking, 648). Kepler’s extensive research led to his third law of planetary motion: the cubes of the mean distance of the planets from the sun are proportional to the squares of their periods of revolution. Referring to table 1, one can infer that the farther a planet is from the sun, the slower it travels in its orbit, and conversely, the closer a planet is to the sun, the faster its orbit around the sun.

**Fig. 15: Planetary Data and Kepler’s Third Law**
Kepler’s harmonic law was his most exciting discovery, and it gave him more reason to believe in a divine creator – the one who, in Kepler’s cosmological vision, set forth the mathematical harmonies of the world. This served as mathematical proof of God’s purposeful design in Nature. In Book 5 of *Harmonies of the World* he writes:

“I dare frankly to confess that I have stolen the golden vessel of the Egyptians to build a tabernacle for my God far from the bounds of Egypt. If you pardon me, I shall rejoice; if you reproach me, I shall endure. The die is cast, and I am writing the book, to be read either now or by posterity, it matters not. It can wait a century for a reader, as God himself has waited six thousand years for a witness.” (Kepler in Hawking, 633)

Kepler’s planetary observations did not automatically earn them the title of “Laws” of motion as his ideas were initially received with skepticism (Smith, 2008). This is understandable, since his new view of the universe, together with Galileo’s telescopic observations, broke away from a long standing scholastic tradition – a centuries-old tradition that held Aristotle as having the final word on matters of natural knowledge. Kepler was also accused for incorporating mysticism – unfounded, fabricated facts about natural phenomena – in his theories, which did not sit well with the greater scientific community. The non-scientific aspect of his publication *Harmonices Mundi* was in the way he tried to fit the numerical data on planetary motion and location with the table of contents in Book V (Cohen, 145):

1. Concerning the five regular solid figures.
2. On the kinship between them and the harmonic ratios
3. Summary of astronomical doctrine necessary for contemplation of the celestial harmonies
4. In what things pertaining to the planetary movements the simple harmonies have been expressed and that all those harmonies which are present in song are found in the heavens.
5. That the clefs of musical scale, or pitches of the system, and the kinds of harmonies, the major and the minor, are expressed by certain movements

Additionally, despite Kepler’s great accuracy in his measurements, many contemporary astronomers found his non-circular orbits and his new techniques for calculating planetary velocities too strange for immediate acceptance (Kuhn, 225). Luckily for Kepler, his work did
not go unnoticed. There was ample evidence, in Brahe’s extensive astronomical charts, to suggest that planets moved in ellipses (at varying speeds) and not in perfect circles (in uniform motion) around the sun (as Copernicus had shown).

Kepler’s second law of planetary motion suggested the existence of a dynamic, universal mechanism that would account for the varying speeds of the planets. Kepler had conceptualized this phenomenon, or source of planetary movement, as coming from a solar force – different from the inverse square law obeyed by solar light, which emanated in all directions – that acted only along the plane of the ecliptic (i.e. the relatively flat plane in which the planets orbit around the sun) (Cohen, 144). His reasoning was that this anima motrix only functioned to move the planets in their orbits. He did not see any reason why this force should emanate in all directions. Here is evidence that Kepler sometimes fashioned his theories according to his own interpretations. Kepler could not provide a scientific reason for this phenomenon. Isaac Newton, an established English scientist, would find inspiration in his research and carry out a scientific program, based on deductive reasoning, that would describe the mathematical law underpinning Kepler’s newly founded elliptical trajectory.

Isaac Newton’s Vision and Method

When Newton first reviewed Kepler’s results, he realized that there was a missing puzzle in Kepler’s explanation for planetary orbits. Newton noticed that Kepler’s three laws were mutually independent in that the three laws were not made to follow one another in a logical manner. This does not, in any way, discredit Kepler’s findings, but it speaks to the caliber of Newton’s scientific mind. Newton was after a grand unified theory of physics, one that would unite terrestrial with celestial dynamics. Newton had embedded within his own scientific program four important rules that would guide his research to find the fundamental laws of Nature. He called these the Rules of Reasoning in Philosophy, which states:

(1) Rule I: We are to admit no more causes of natural things than such are both true and sufficient to explain their appearances;

(2) Rule II: Therefore to the same natural effects we must, as far as possible, assign the same causes;

(3) Rule III: The Qualities of bodies, which admit neither intension nor remission of degrees, and which are found to belong to all bodies within the reach of our experiments, are to be esteemed the universal qualities of all bodies whatsoever; and

(4) Rule IV: In experimental philosophy we are to look upon propositions collected by general induction from phenomena as accurately or very nearly true, not withstanding any contrary hypotheses that may be imagined, till such time as other phenomena occur, by which they may either be made more accurate, or liable in exceptions
In summary, Newton informs the reader that: (1) if a cause, for the appearance of Natural phenomena, is found to be both sufficient and true, then additional (i.e. unnecessary) causes can be discarded; (2) we can assign the same causes for a wide array of phenomena that display the same natural effects; (3) the properties of matter that can be readily experimented on applies to all matter in the universe; and (4) the data we collect from observed phenomena, through inductive investigation, cannot be denied, nor replaced, with other hypothesis, unless the observed phenomena in question is found, at a later time, to deviate from data previously collected.

Reading more carefully, Rule I reflects Newton’s philosophical stance: that Nature “does nothing in vain, [and] more is in vain when less will serve; for nature is pleased with simplicity, and effects not the pomp of superfluous causes” (Newton in Hawking, 1038). Newton firmly believed that the most basic, fundamental laws of Nature were simple and that the formulation of this law can be described with mathematical economy/simplicity – Galileo had modelled this scientific ideal in his study of falling bodies when he demonstrated a simple relationship between time and distance (i.e. \(d \propto t^2\)). Rule II says that the mechanism for breathing in humans should be the same for other animals, and that the dynamics of falling bodies in Europe should be the same for falling bodies in America. Rule II suggests that the principles of motion in the celestial realm can be explained with the same principles of motion in the terrestrial realm (as below, so above). Rule III is an extension of Rule II in that he explains the moon’s gravitation toward the earth in much the same way as “all bodies about the Earth gravitate towards the Earth...in proportion to the quantity of matter which they severally contain” (Newton in Hawking 1039-1040). Rule IV is self-explanatory: if a planet is observed to travel at a certain speed across the sky, then hypotheses that suggest reasons for a different orbital speed must be rejected – we cannot deny inductive arguments with hypothesis that contradict the observed data.

In the context of Keplerian orbits, Newton was after the cause for elliptical orbits, a simple mathematical property that would describe the nature of the force that held the planets in their orbits. In search for the answer, Newton asked an important question in his synthesis: Are the fundamental dynamical laws that govern both terrestrial and celestial motion one and the same? This question suggests a grand unified theory of physics, which faithfully follows Newton’s scientific program (Rules of Reasoning in Philosophy) set out in Book III of the Principia. This acts as the philosophical foundation for Newton’s monumental work in deriving Kepler’s three laws of planetary motion and in formulating his Universal Law of Gravitation.

The Rules of Reasoning in Philosophy demonstrates Newton’s professional quality as a highly-disciplined scientist. Newton’s reaction to Kepler’s work reflects a belief widely held
among the theological scientists of his time – the idea that God had created the world under one unified principle. Newton was not satisfied with mere speculation, especially philosophies that were disconnected from empirical evidence. Newton was very careful not to reduce his efforts to the realm of fantasy and fiction, “We are certainly not to relinquish the evidence of experiments for the sake of dreams and vain fictions of our own devising;…” (Newton in Hawking, 1039). Newton saw the great power and predictive value in Science. As such, Newton was after the fundamental laws of nature that would explain, and hence unify, Kepler’s findings. Newton set off to find the reason why the planets would follow in elliptical orbits.

Newton’s Principia, Proof of Elliptical Orbits

It wasn’t until Newton’s famous publication, the Philosophiæ Naturalis Principia Mathematica (1687), considered one of the most important works in the entire history of the physical sciences, that Kepler’s theoretical speculations were confirmed. The Principia was divided into three books: (1) Book I dealt with the motion of bodies in the heavens, assuming the absence of resistance; (2) Book II applied mechanical principles, for the first time, to solve problems of motions in resisting media; and (3) Book III addressed more specific problems in the motions of celestial bodies, including mathematical demonstrations for the nature of tides, comets and other phenomena in the cosmos. This report will look more closely at the result found in Books I and II of the Principia as they pertain to planetary motion.

In the Principia, Newton applied analytical techniques, in the style of ancient Greek geometry, using geometrical proofs throughout (see fig. 16, 17), and proved Kepler’s planetary laws as a direct consequence of his mathematical demonstrations (Segre, 61).

Fig. 16: Excerpt from Newton’s Principia; The Invention of Centripetal Forces, Proposition I Theorem I
Fig. 17: Excerpts from Newton’s Principia; *On the Motion of Bodies in Eccentric Conic Sections*, Proposition XL Theorem VI

Note: In figures 16 and 17, Newton explains his reasoning in monologue style writing, while occasionally providing reference to his diagrams. These pages are difficult to follow as he skips the proofs for most of the mathematical details in his explanation. This might explain how very few scientists, upon publication of the Principia, were able to understand the full scope of Newton’s mathematical reasoning. Newton had told a friend “to avoid being bated by little smatterers in mathematics I designedly my principle abstruse; but yet so as to be understood by able mathematicians” (Segre, 61).

But how did Newton go about doing this? Newton was tactful in orchestrating his methods into a coherent and logically sound scientific program. Isaac Newton had unified his *Three Laws of Terrestrial Motion* with Galileo’s findings on *Inertia*, including the application of *Fluxions*, to derive the mathematical rule that would describe the centripetal force that maintained the planets in their elliptical orbits. He called this the *Universal Law of Gravitation*, a force that he explicitly states in Book III (Proposition II, Theorem II) of the *Principia*.

Not only did Newton confirm Kepler’s speculations, but he also proposed a physical theory of gravity that addressed the question of *why* the planets moved in ellipses – a major feat which Kepler was not able to prove in his model. Where Kepler had provided the
kinematics of planetary motion, Newton demonstrated the dynamical principles that would give rise to Keplerian elliptical motion.

**Newton’s Mathematical Reasoning**

Newton begins his grand synthesis by assuming that planets move in elliptical orbits. That is, he assumes Kepler’s first law of planetary motion to be true. He does this without violating his *Rules of Reasoning in Philosophy* as the data was shown, by Kepler, to accurately account for astronomical observations. Here is a flowchart for each step that Newton takes in preparing and finalizing his mathematical proof of Kepler’s three laws of planetary motion:

1. Newton assumes elliptical orbits
2. Newton explains that a curved orbit is caused by a center seeking, centripetal force towards the sun
3. Newton proves Kepler’s area law (2nd law), using Galileo’s Law of Inertia (or Newton’s first law of motion) and Fluxions
4. Newton applies geometry and fluxions to demonstrate that the center seeking force towards the sun follows an inverse square law

I will explain the details in each of the four steps that Newton takes in proving Kepler’s results. Step four is where Newton deviates from Kepler and provides the mathematical description of the force that keeps planets in their elliptical orbits around the sun.

**(1) Newton Assumes Elliptical Orbits (see fig. 18)**

Newton has reason to begin with this assumption as it follows Rule IV in Rules for Reasoning in Philosophy – because one cannot deny inductive observations, of celestial phenomena, that are consistent with the data collected (in Brahe’s astronomical manuals).

**Fig. 18: Excerpt from Newton’s Principia; On the Motion of Bodies in Eccentric Conic Sections, Proposition XL Theorem VI**
To prepare the stage for the grand proof, he must first show that there is a centripetal force that pulls the planets into an orbit around the sun. He must also demonstrate that the planet will sweep out equal areas in equal times (Kepler’s 2nd Law).

(2) Newton explains that a curved orbit is caused by a center seeking, centripetal force towards the sun (see Fig. 19)

Fig. 19: Excerpt from Newton’s Principia; Book I, Proposition II Theorem II (with added arrows showing impulse forces acting at points B-C-D of a planet’s orbit around the sun (S))

Newton must first show that there exists a center seeking force (i.e. centripetal force) towards the sun for a sun-planet system. Proposition II Theorem II states: Every body that moves in any curved line described in a plane, and by a radius, drawn to a point either immovable, or moving forward with n uniform rectilinear motion, describes about that point areas proportional to the times, is urged by a centripetal force directed to that point (Newton in Hawking, 767). What Newton does is he says: suppose a planet, at point A, moving in uniform rectilinear motion, moves towards point B. Now, in the absence of any external forces, the planet would continue,
in uniform rectilinear motion, towards point c (by the law of Inertia). This is obviously not the course for the planet as the planet would fly off into space, at constant velocity, along the same line. Newton now introduces an impulse force at point B along the planet’s path, for some time t. Newton reasons, using vectors, that if a center seeking force had acted on the planet at point B, then the planet would experience both a change in velocity and direction. Resolving the vectors, the planet would now follow a path towards C. The magnitude of the force can be measured by the change in distance from its inertial path (i.e. segment Cc). By the same reasoning, if the planet, which is now at point C, were to continue with uniform rectilinear motion, it would continue its trajectory towards point d. Newton adds another impulse force at point C that draws the planet from its rectilinear path. This force, when resolved, leads the planet towards the point D, where the magnitude of the force can be measured by the change in distance from its inertial path (i.e. segment Dd). The way Newton accounts for curved orbits is he says: suppose we diminish, ad infinitum, the breadths of line segments AB, BC, CD, DE, EF, etc., then (by Corollary. IV, Lemma III) their ultimate perimeter ADF will be a curved line (Newton in Hawking, 765-766). This proves that a central seeking force towards the sun would trace out a curved orbital path.

(3) Newton proves Kepler’s area law (2nd law), using Galileo’s Law of Inertia (or Newton’s first law of motion)

Fig. 20: Excerpt from Newton’s Principia; Book I, *Proposition I Theorem I*
In this step (3) of Newton’s reasoning, he shows that Kepler’s area law follows directly from the Principle of Inertia and his application of fluxions on geometry. Proposition I Theorem I states:

*The areas, which revolving bodies describe by radii drawn to an immovable center of force do lie in the same immovable planes, and are proportional to the times in which they are described.*

Newton says: suppose that time is divided into equal parts and that in the first part, being along the planetary path \( \text{A-B} \), the planet moves in uniform rectilinear motion from point \( \text{A} \) toward point \( \text{B} \). In the absence of an external force, the planet, due to its own inertia, will continue moving with uniform rectilinear motion past point \( \text{B} \) towards point \( \text{c} \). Since the time interval between segment \( \text{Bc} \) is equal to that of segment \( \text{AB} \), then the areas of triangles \( \text{SAB} \) and \( \text{SBc} \) are equal. Newton uses the following geometrical proof to show that \( \text{area triangle SAB} = \text{area triangle SBc} \), similar to Galileo’s study on uniform rectilinear motion (see fig. 21):

**Fig. 21: Galileo’s Study of Uniform Rectilinear Motion, Equal Areas in Equal Times Law**

In figure 21, Galileo allows a body to travel in uniform, rectilinear motion from point \( \text{A} \) towards points \( \text{B-C-D-E} \), if left uninterrupted by an external force. The spacing of each segment \( \text{AB-BC-CD-DE} \) are made to be equal in time. Galileo reasons that the area of triangle \( \text{SBC} \) is equal to the area of triangle \( \text{SCD} \) as both triangles share the same height and base (i.e. time interval, \( \text{BC} = \text{CD} \)) (see fig. 22).
Newton now needs to show (see fig. 20) that the area of triangle $SAB$ is equal to the area of triangle $SBC$. The way Newton does this is by showing that if $\text{area triangle } SCD = \text{area triangle } SBC$, then $\text{area triangle } SAB = \text{area triangle } SBC$. If this logic is continued for all other triangles formed by the orbit around the sun, then Kepler’s $2^{nd}$ law holds true (that a planet moving in an ellipse around the sun sweeps out equal areas in equal times). Recall that a planet’s curved orbit around the sun $S$ is necessitated by a centripetal, center-seeking force towards the point $S$, and that this force acts in impulses specifically at points $A, B, C, D, E, F$, etc. Newton draws the planet’s displacement segment $Cc$ parallel to the segment $SB$, which is the same line along which the first impulse force acts is applied (see fig. 19). Why would Newton draw segment $Cc$ parallel to $SB$? The reason for this is that if we consider a small differential of time, $dt$, after point $B$ (or if we make point $C$ come infinitely close to point $B$), we can say that the impulse force along segment $Cc$ becomes parallel to the impulse force acting on the planet along segment $SB$ (see fig. 23). Let’s consider the triangle formed by $SBC$. 

Fig. 22: Newton’s Demonstration of Equal Areas in Equal Time, For Uniform Rectilinear Motion
Here we will prove the following statement: Joining S and C, given $SB$ is parallel to $Cc$, the triangle $SCB$ will be equal to the triangle $SBC$ and hence also to the triangle $SAB$. Proving this statement leads us to the following argument: Since triangles $SBC$ and $SBC$ have the same base, and since the deviation segment $Cc$ (i.e. from uniform rectilinear motion) is parallel to the line of force along segment $SB$, they will have the same perpendicular, falling to the extension of segment $SB$ (therefore the height of segment $CC' = \text{the height of segment } cc'$). Thus, the areas of triangles $SBC$ and $SBC$ are equal because they have a common base and equal heights. If we follows the same argument, if the impulse force, seeking point $S$, were to act successively at points $C, D, E$, thereby creating a planetary path along $CD, DE, EF, etc.$ then the area of triangle $SCD$ will be equal to triangle $SBC$, $SDE$ to $SCD$, $SEF$ to $SEF$, etc. (Brackenridge, 84).

(4) Newton applies geometry and fluxions to demonstrate that the center seeking force towards the sun follows an inverse square law

In this section, we will see how Newton prepares the grounds for determining the mathematical law for the centripetal force that keeps the planets in their elliptical orbits. Here is a summary of some key mathematical results from both Galileo and Kepler, key pieces to Newton’s synthesis:

<table>
<thead>
<tr>
<th>Galileo</th>
<th>Since $d \propto at^2 \Rightarrow a \propto \frac{d}{t^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kepler</td>
<td>Area $\propto$ Time $\Rightarrow$ $Area \propto t$</td>
</tr>
</tbody>
</table>
Galileo concluded that distance of falling bodies, under constant acceleration, was proportional to the square of each time interval. Re-arranging this proportion gives us a proportion for acceleration that is inversely proportional to time squared. The reason why this relationship is important is that both Galileo and Newton knew that acceleration was due to the force of gravity, and that to describe this force, one must set up the correct proportionality to proceed. Kepler made the important observation that the area swept out by the planet is proportional to the time elapsed. Newton combines both results together in a synthesis of ideas about the fundamental laws of Nature:

**Newton** Since \( F = ma \), then \( F \propto a \) and, from Galileo and Kepler, we can say that

\[
F \propto a \propto \frac{d}{t^2} \propto \frac{d}{\text{Area}^2}
\]

From Newton’s Axioms, or Laws of Motion, he states in Law II: *The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed* (Newton in Hawking, 743). Here he states that force is directly proportional to acceleration, leading to his equation \( F = (\text{mass})(\text{acceleration}) = ma \). Since Newton finds a direct proportion between force and acceleration, he extends Galileo’s result and hypothesizes a connection between terrestrial and celestial gravity. Together with Kepler’s 2nd law, Newton shows, in Proposition VI Theorem V, that the nature of the universal gravitational force to be directly proportional to distance and inversely proportional to the square of the time (Newton interchanges time with area in a later argument). Armed with this proportionality, Newton continues to make mathematical deductions until he gets to the fundamental truth about this force (see fig. 23).
Suppose that a planet at $P$ moves along a curve towards $Q$, under the influence of a centripetal force towards the sun. Newton makes the following mathematical observation:

$$F \propto a \propto \frac{d}{t^2} \propto \frac{QR}{Area^2}$$

The distance term, $d$, in the proportion, corresponds to the displacement segment $QR$ in fig. 23 as a centripetal force acts to pull the planet away from its inertial path $PR$ (when there is no force present in the system). Hence, the strength of the gravitational force towards $S$ is directly proportional to distance that the planet has been displaced, or $QR$. Combining this with Kepler’s observation that time is directly proportional to area, Newton finds the correct relationship to begin his proof. Note that we haven’t clearly defined what the $Area^2$ term corresponds to in fig 23, but one can guess that it relates to kepler’s 2nd law or equal areas in equal times. In fig. 24, we see more details added to the proof.
Newton then drops a perpendicular segment QT onto Segment SP applies Kepler’s second law to come up with the following proportion:

\[ F \propto \frac{QR}{A_{SPQ}}^2 \propto \frac{QR}{\left(\frac{1}{2} SP \cdot QT\right)^2} \]

In this proportion he equates the area\(^2\) term with the area of the triangle, namely triangle SPQ, swept out by the planet. The perpendicular segment QT to SP acts as the height of triangle SPQ, and therefore writes the area of triangle SPQ as \(\left(\frac{1}{2} SP \cdot QT\right)^2\). This leads to the following proportion, eliminating the constant terms:

\[ F \propto \frac{QR}{(SP \cdot QT)^2} \]

\[ F \propto \frac{QR}{SP^2 \cdot QT^2} \]
Newton’s goal is to show that the ratio \( \frac{QR}{QT^2} \rightarrow 1 \) (by moving the point Q infinitely close to point P) thereby showing that the centripetal force towards the sun is inversely proportional to the square of the distance \((SP)\) from the sun to the planet:

\[
F \propto \frac{1}{SP^2}
\]

In Section III (On The Motion Of Bodies In Eccentric Conic Sections) of Book I of the *Principia*, Newton prepares to demonstrate that \( \frac{QR}{QT^2} \rightarrow 1 \) (see fig. 25).

**Fig. 25: ON THE MOTION OF BODIES IN ECCENTRIC CONIC SECTIONS**

*Proposition XL, Problem VI*

Newton’s proposition states: *Of a body revolves in ellipses; it is required to find the law of the centripetal force tending to the focus of the ellipse* (Newton in Hawking, 778).

The geometrical details in figure. 25 can be easily shown in figure.26 (below). Here Newton includes both the semi-major \((CA)\) and semi-minor \((CB)\) axes, including the latus rectum (see fig. 26) passing through the foci point \(S\) where the sun is situated.
Applying the properties of ellipses, Newton writes the following equality:

$$\frac{L \cdot QR}{QT^2} = \left[ \frac{2CP}{GV} \right] \left[ \frac{QV^2}{QX^2} \right]$$

In the equation above, $L$ refers to the length of the latus rectum passing through the foci point $S$. The segment $QT$ is perpendicular to segment $SP$. Segments $QV$ (point $V$ being the extension of segment $QX$ onto segment $CP$) and $QX$ run parallel to segment $RP$. This equation maintains equality as point $Q$ is dragged to any point along the elliptical path around $S$. Additionally, as point $Q \to P$ (i.e. point $Q$ moves infinitely close to point $P$), segment $QV = segment QX$, as they become one in the same line. Also, *segment GV reaches, in its limit, segment 2CP*. Recall Newton’s proportionality for the centripetal force directed towards the sun:

$$F \propto \frac{QR}{SP^2 \cdot QT^2}$$
Newton refers to an earlier proportion to show that the ratio $\frac{QR}{QT^2} \to 1$. In this case, he applies properties of the ellipse, drawing from Apollonius’ *Conics*:

$$\frac{L \cdot QR}{QT^2} = \left[ \frac{2CP}{GV} \right] \left[ \frac{QV^2}{QX^2} \right]$$

Newton allows for the point $Q$ to come infinitely close to point $P$ (see fig. 26), and shows that the ratios $\frac{2CP}{GV}$ and $\frac{QV^2}{QX^2}$ both approach unity:

$$\left[ \frac{2CP}{GV} \right] \to 1 \text{ and } \left[ \frac{QV^2}{QX^2} \right] \to 1$$

This result of this limit (drawing the point $Q$ infinitely close to point $P$) implies that

$$\frac{L \cdot QR}{QT^2} = 1$$

Finally, this shows that the mathematical description of the centripetal force that keeps the planets in their elliptical orbits follows the proportion:

$$F \propto \frac{1}{SP^2}$$

Newton’s proof is far more detailed than what has been presented here. I am excluding from this report the many steps Newton took, using properties of ellipses found in Apollonius’ *Conics*, to prove, in the style of Greek geometry, the following result:

$$\frac{L \cdot QR}{QT^2} = 1$$

A more detailed explanation can be found in Bruce Brackenbridge’s book, *The Key to Newton’s Dynamics*. 
Conclusion

The combined efforts of Galileo, Brahe, Kepler, and Newton reinforces the ancient Greek idea that the universe could be understood with the human mind. We learn from these monumental figures that proper science requires: ingenuity and logic, hard work, method, determination, skill, discipline, questioning even the most obvious facts, dissemination of ideas, collaboration, and a relentless pursuit to find the most fundamental truths about the universe. Unfortunately, progress towards these ends has been hindered by supposing mystical causes for observed phenomena. The collective effort in establishing everlasting truth meant that we had to suspend our beliefs and suppositions about the world and begin with the most basic physical facts, and build up our knowledge about the world in small increments. Newton once said, “To explain all nature is too difficult a task for any one man or even for any one age. 'Tis much better to do a little with certainty & leave the rest for others that come after than to explain all things by conjecture without making sure of any thing.” Even in ounce of truth is better than a slew of unfounded ideas and hypotheses. It was after many centuries, and a major break in the traditional belief system that the universe could be understood more clearly through careful astronomical observation, data analysis, and mathematical rigor. It took many centuries for a more sophisticated, scientific understanding of the universe – one that extended and applied the fundamental principles of motion on Earth to accurately explain celestial dynamics. Isaac Newton realized that the motion of objects in the heavens, such as planets, the Sun, and the Moon, and the motion of objects on the ground, like cannon balls and falling apples, could be described by the same set of physical laws.

Isaac Newton, is widely considered the most famous scientist of the 17th century. The publication of his *Principia* (1658) was testament of his profound ingenuity. The *Principia* provided the first mathematically sound explanation for the unification of terrestrial and celestial dynamics. Contrasting with Galileo and Kepler, Newton did not limit his professionalism in science to fit theories with observational data. He built the foundations of his knowledge using the most concrete facts of physics, and deduced, from these facts, properties about the universe that could not be seen with the naked eye (i.e. universal gravitational force). Newton dealt with the mathematical objects themselves – applying both inductive and deductive reasoning to derive new facts – to prove the true nature of planetary motion. This is the true power of science, propelled by our passions to understand even the smallest truths that the universe holds hidden behind the veil.
References